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# Super Janus

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Abstract: We propose and study a supersymmetric version of the Janus domain wall solution of type-IIB supergravity. Janus is dual to $\mathcal{N} = 4$ super Yang Mills theory with a coupling constant that jumps across an interface. While the interface in the Janus field theory completely breaks all supersymmetries, it was found earlier that some supersymmetry can be restored in the field theory at the cost of breaking the SO(6) R-symmetry down to at least SU(3). We find the gravity dual to this supersymmetric interface theory by studying the SU(3) invariant subsector of $\mathcal{N} = 8$ gauged supergravity in 5d, which is described by 5D $\mathcal{N} = 2$ gauged supergravity with one hypermultiplet.

Keywords: AdS-CFT and dS-CFT Correspondence.



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JHEP10(2005)094

## Contents



## 1. Introduction

Recently, a domain wall solution to type-IIB supergravity was discovered which explicitly breaks supersymmetry while retaining stability [1]. This solution, dubbed "Janus" after the Roman god with two faces, includes a dilaton and a 5-form. The dilaton develops a kink profile in this solution, approaching different boundary values on either side of the domain wall (hence, two faces). Non-perturbative stability was proven for Janus-type solutions in $\mathrm{AdS}_d$ in [2]. This extends to IIB with the mild assumption that KK modes on the $S_5$ factor of the geometry do not destabilize the solution. In [3], a gauge theory was proposed and investigated as an AdS/CFT dual of this solution. This gauge theory is ordinary $\mathcal{N} = 4$ Yang-Mills with different gauge couplings on either side of a defect (or interface) that the boundary theory inherits from the bulk domain wall. It was found that in the field theory, partial SUSY could be restored by the addition of certain counter-terms localized to the defect at the cost of breaking the R-symmetry at least as far as SU(3).

We now propose a supergravity dual of this field theory, which we dub Super Janus. The uplift formulas of [4] allow one to obtain solutions of IIB SUGRA in 10D from solutions of $\mathcal{N} = 8$ gauged SUGRA in 5D. We restrict ourselves to the subsector of this theory which is invariant under the SU(3) R-symmetry of the field theory. Under this restriction we find 4 scalars. This subsector is well described by gauged $\mathcal{N} = 2$ SUGRA with one hypermultiplet, where there is well-known formalism for solving the equations of motion for supersymmetric domain walls [6 – 8].



JHEP10(2005)094

## 2. Review of Janus

### 2.1 Janus itself

The Janus ansatz is an AdS$_4$-sliced domain wall supported by a dilaton and 5-form:

$$
\begin{aligned}
ds_{10}^2 &= e^{2U(\mu)}\left(g_{ij}dx^i dx^j + d\mu^2\right) + ds_{S^5}^2 &(2.1)\\
\phi &= \phi(\mu) &(2.2)\\
F_5 &= 4\left(e^{5U(\mu)}d\mu\wedge\omega_{AdS_4} + \omega_{S^5}\right), &(2.3)
\end{aligned}
$$

where $g_{ij}$ is the metric of AdS$_4$ with unit scale in an arbitrary slicing. With this ansatz, the IIB SUGRA equations of motion can be simplified to the following two equations:

$$
\begin{aligned}
\phi'(\mu) &= c\, e^{-3U(\mu)} &(2.4)\\
U'(\mu) &= \sqrt{e^{2U} - 1 + \frac{c^2}{24}\, e^{-6U}}\,, &(2.5)
\end{aligned}
$$

where the constant, $c$, can be thought of as an (arbitrary) integration constant arising from the dilaton's second order equation of motion:

$$
\partial_\mu\left(\sqrt{g}g^{\mu\nu}\partial_\nu\phi\right) = 0\,. \tag{2.6}
$$

The largest root (in terms of $U$) of the equation $U'(\mu) = 0$ then determines the value of the warp factor on the domain wall, $U_0$, and the range of the angular coordinate $\mu$ is determined by integrating equation (2.5):

$$
\mu_0 = \int_{U_0}^{\infty} dU \sqrt{e^{2U} - 1 + \frac{c^2}{24}\, e^{-6U}}\,. \tag{2.7}
$$

Note that undeformed $AdS_5$ has $\mu_0 = \pi/2$ while Janus has $\mu_0 > \pi/2$. There is a critical value of $c$ above which the geometry becomes singular:

$$
c_{cr} = \frac{9}{4\sqrt{2}}\,. \tag{2.8}
$$

For $c \geq c_{cr}$, the zeros (in $U$) of equation (2.5) become complex. When this happens, it becomes impossible for the warp factor to have a turning point at the wall. Instead, the warp factor must be allowed to grow arbitrarily negative as you approach the wall, and the geometry becomes nakedly singular. This is discussed in greater detail in [1−3, 5].

An alternative interpretation is that one is free to pick any negative value for $U_0$ (within a range we will see shortly). The constant $c$ is then determined by the vanishing of equation (2.5):

$$
c = \sqrt{24e^{6U_0}(1 - e^{2U_0})}\,, \tag{2.9}
$$

and the value of $\mu_0$ is determined as before. From this perspective, as we decrease $U_0$ from zero, we find that $c$ increases rapidly from zero to a maximum of $c_{cr}$ at $U_0 = -\ln(2/\sqrt{3})$. We must stop there, as we are interested in the *largest* root of equation (2.5). It would



JHEP10(2005)094

not do for the warp factor to have two turning points, so we must restrict our choice to $U_0 \in (-\ln(2/\sqrt{3}), 0)$. While choosing the value of $c$ directly is more convenient for the study of Janus and its dual, when studying Super Janus it becomes more convenient to specify the behavior of the warp factor on the domain wall (for numerical integration purposes). We will find Super Janus possesses an even narrower critical range for the values of $U_0$. The nature of this critical range of parameters in Super Janus appears to be somewhat different from the critical range we find in Janus. It is also interesting to note that there is no obvious interpretation of this criticality in the dual of either theory. In a gauge theory with jumping coupling, there is no obvious reason why the absolute amount of the jump in coupling strength should cause a breakdown of any of the salient features required for AdS/CFT duality. We postpone deeper study of this for the future.

### 2.2 The Gauge theory dual of Janus

The boundary theory dual to Janus was constructed and examined in detail in [3]. The essential point is that the lagrangian is simply that of $\mathcal{N} = 4$ Yang-Mills with a jump in the gauge coupling at the defect inherited from the bulk. Spatial dependence of the gauge coupling means integration by parts is no longer trivial, and consistency requires that one write the scalar field kinetic term as $\frac{1}{2} X^I D^\mu D_\mu X^I$, where $D_\mu X^I = \partial_\mu X^I + i[A_\mu, X^I]$. If we denote by $\mathcal{L}'$ the lagrangian for $\mathcal{N} = 4$ Yang-Mills with this modified scalar kinetic term normalized with the gauge coupling appearing only as $1/g^2$, we may write the action for the gauge theory dual of Janus as follows:

$$S_{\text{Janus}} = \int d^4x \, (1 - \gamma \varepsilon(x_3)) \, \mathcal{L}'(x) \,, \tag{2.10}$$

where $\varepsilon$ is an odd step function, $\mathcal{L}'$ is understood to have the average gauge coupling $(g^{-2} = \bar{g}^{-2} = \frac{1}{2}(g_+^{-2} + g_-^{-2})$, and

$$\gamma = \frac{g_+^2 - g_-^2}{g_+^2 + g_-^2} \,. \tag{2.11}$$

The $\pm$ subscripts refer to the sign of $x_3$, i.e. on which side of the defect one is located.

### 2.3 Restoring SUSY to the field theory dual of Janus

In this subsection we briefly review the construction of partial interface SUSY from appendix A of [3]. This is the field theory for which we will then construct a gravitational dual. In [3] this construction was used as a proof that it was not possible for partial supersymmetry to creep back into the boundary gauge theory dual of the explicitly non-supersymmetric Janus solution. The construction does, however, produce a perfectly good, if somewhat peculiar, gauge theory.

The first step is to consider the $\mathcal{N} = 4$ fields as being made up of one $\mathcal{N} = 1$ vector multiplet and $3\,\mathcal{N} = 1$ chiral multiplets. One then considers how the SUSY variations of the $\mathcal{N} = 1$ lagrangians are modified by the inclusion of a Janus-style spatially varying coupling constant (varying only in the $x_3$ direction which we will now denote as $z$). In practice, one treats the coupling as a continuous function and takes the limit as it approaches a step function. The chiral lagrangian we consider is

$$\mathcal{L} = -\partial_\mu\phi^*\partial^\mu\phi - \frac{i}{2}\bar{\psi}\Gamma^\mu\partial_\mu\psi + F^*F + W'F - \frac{i}{2}W''\bar{\psi}P_+\psi\,, \tag{2.12}$$

where $P_\pm = (1 \pm \Gamma^5)$, and all dependence on the spatially varying coupling constant is assumed to be in the superpotential, $W$. When considering the SUSY variation of this lagrangian, one finds that there are now terms which cannot be written as a total derivative:

$$\delta\mathcal{L} \supset -i\sqrt{2}\,\partial_z g\,\bar{\varepsilon}\left(P_+\Gamma^z\frac{\delta W'^*}{\delta g} + P_-\Gamma^z\frac{\delta W'}{\delta g}\right)\psi\,. \tag{2.13}$$

Now suppose that we preserve 2 supercharges in the general spirit of supersymmetric defect conformal field theories [14]. Using the projector condition $\Pi\varepsilon = \varepsilon$, where

$$\Pi = \frac{1 + i\Gamma^5\Gamma^z}{2}\,, \tag{2.14}$$

the offending terms in $\delta\mathcal{L}$ (2.13) can be rewritten as

$$\sqrt{2}\,\partial_z g\,\bar{\varepsilon}\left(-P_-\frac{\delta W'^*}{\delta g} + P_+\frac{\delta W'}{\delta g}\right)\psi\,. \tag{2.15}$$

This expression happens to be a SUSY variation of $2\,\partial_z g\,\mathrm{Im}\frac{\delta W}{\delta g}$. Thus, we may restore partial susy to the chiral sector by subtracting this term from the lagrangian, effectively adding a counterterm. This counterterm takes the form of a delta function in the limit where $g$ becomes a true step function, so we say the counterterm is localized on the defect. Interestingly, this prescription fails if one normalizes the lagrangian such that the coupling appears as an overall $1/g^2$ or if one writes the scalar kinetic term as $\phi^*\Box\phi$.

For the vector multiplet, the analogous prescription fails for the "opposite" normalization where we normalize to put $g$ in the numerator. We are forced to normalize the vector lagrangian as follows:

$$\mathcal{L} = -\frac{1}{4\,g^2}F^a_{\mu\nu}F^{a\mu\nu} - \frac{i}{2\,g^2}\bar{\lambda}^a\Gamma^\mu D_\mu\lambda^a + \frac{1}{2\,g^2}D^aD^a\,. \tag{2.16}$$

The offending terms in the SUSY variation of this lagrangian may be written as a supersymmetric variation of $\partial_z(1/4g^2)\bar{\lambda}^a\Gamma^5\lambda^a$, after again imposing the projector condition $\Pi\varepsilon = \varepsilon$, with $\Pi$ given in (2.14).

Attempting to reassemble these lagrangians into an $\mathcal{N} = 4$ multiplet one finds no further obstruction to supersymmetry from gauge covariant derivatives and Yukawa interactions. However, the counterterms treat the multiplets very differently, and, thus, the $\mathcal{N} = 1$ gaugino can no longer be mixed with the fermions from the chiral multiplets. Therefore, the maximal R-symmetry such a theory may possess is SU(3). In the next section we construct the gravity dual of the theory with this maximal R-symmetry. Note that the boundary gauge theory only has two supercharges but still possesses defect conformal symmetry, so the SUSY preserved by the dual gravity theory will be $\mathcal{N} = 1$.



JHEP10(2005)094

## 3. $\mathcal{N} = 8$ gauged SUGRA in 5D

The $\mathcal{N} = 8$ gauged SUGRA theory has 42 scalars which comprise an $E_{6(6)}/\mathrm{USp}(8)$ coset. These 42 scalars can be organized in terms of the SO(6) R-symmetry of $\mathcal{N} = 4$ Yang-Mills by looking at the symmetric traceless component of various operators. Scalar masses provide $6 \times 6 \to (20' + 1)_s + 15_a$ which gives us 20 traceless scalars. Fermion masses provide $4 \times 4 \to 10_s + 6_a$, and $\bar{4} \times \bar{4} \to \overline{10}_s + \bar{6}_a$, which gives us a 10 and $\overline{10}$, for 20 more scalars. The axion and dilaton round out our set of 42. Next, we truncate this to the SU(3) invariant subsector. This sector of the theory has been extensively studied in [4, 15]. The breaking pattern of our scalar reps as SO(6) breaks to SU(3) is

$$20' \to 6 + \bar{6} + 8 \tag{3.1}$$

$$10 \to 1 + 6 + \bar{3} \tag{3.2}$$

$$\overline{10} \to 1 + \bar{6} + 3\,, \tag{3.3}$$

giving us an additional 2 scalars which are singlets under our R-symmetry. Thus, we find a total of 4 SU(3) invariant scalars.

These 4 scalar fields live on an $\mathrm{SU}(2,1)/\mathrm{SU}(2) \times \mathrm{U}(1)$ coset, and thus they can be naturally assembled into the scalar sector of the universal hypermultiplet of $\mathcal{N} = 2$, 5D SUGRA. For this theory, there exists plentiful machinery for solving the BPS equations [6–8]. Additionally, the work of [6] showed that with only hypermultiplets present, a simple consistency condition guarantees that the BPS equations will solve the equations of motion. We present and solve numerically a supersymmetric domain wall ansatz which is smooth and displays the correct qualitative features to be a gravity dual of the supersymmetric version of the Janus boundary field theory, namely one scalar field develops a kink profile, and the warp factor turns around at the domain wall.

### 3.1 $\mathcal{N}$=2 flow equations

The scalar manifold of the $\mathcal{N} = 2$ theory with a single hypermultiplet has 8 isometries to gauge. Which isometries are gauged determine a triplet of SU(2) Killing pre-potentials which in turn determine the scalar potential and the flow equations for the scalars. The most general form of the 8 Killing vectors and the 8 corresponding pre-potentials are given in [7]. Our theory really lives in IIB in 10D, so we are forced to choose our gauging very carefully in order to match the fixed points of the scalar potential. From the 10D perspective, we expect a 2D plane of critical points where the 10D dilaton and axion can take any value, but the other scalars are fixed. Thus we are forced to pick a gauging for the $\mathcal{N} = 2$ theory that will give us a plane of fixed points. As in the $\mathrm{SU}(2) \times \mathrm{U}(1)$ invariant subsector studied in [7], this requirement uniquely fixes the gauging (up to a global symmetry transformation). Additionally, we may identify the scalars parameterizing this plane as the 5D dilaton and axion. Unfortunately, the uplift formulas generally entangle the 5D dilaton/axion with the metric [4] so that it is not straightforward to read off 10D behavior from 5D or to unambiguously identify 10D fields with 5D counterparts, even though [4] in principle tells us how to uplift. Though complicated, the existence of an



JHEP10(2005)094

uplift is guaranteed by [7], which details an $\mathcal{N}=2$ truncation of the FGPW flow in $\mathcal{N}=8$ 5D SUGRA [13]. The field content and gauge structure of our model can be embedded in that truncation.

We parameterize the universal hypermultiplet with the 4 scalars $V, \sigma, r, \alpha$. This choice can be obtained from the standard parametrization used in [7] by redefining their $\theta, \tau$ fields as $\theta = r\sin\alpha, \tau = r\cos\alpha$. The metric of the scalar manifold is

$$ds^2 = \frac{1}{2V^2}\,dV^2 + \frac{1}{2V^2}\,d\sigma^2 - \frac{2r^2}{V^2}\,d\sigma d\alpha + \frac{2}{V}\,dr^2 + \frac{2r^2}{V}\left(1+\frac{r^2}{V}\right)d\alpha^2\,. \tag{3.4}$$

In the language of [7], the Killing vectors of the quaternionic manifold are reorganized into generators of SU(2,1). Constants $\alpha_i$ denote coefficients of SU(2) generators, and $\beta$ denotes the coefficient of the compact U(1) (we do not consider non-compact gaugings). With only a hypermultiplet, the gauged isometry must live in this $\mathrm{SU}(2)\times\mathrm{U}(1)$ subgroup. Our model differs from the "Toy model with only a hypermultiplet" of [7] only in the slicing of the domain wall, so the gauge and potential structure must be the same. In order to match the fixed point structure of the potential as discussed above, we gauge the isometry corresponding to the choice $\beta = -\alpha_3, \alpha_1 = \alpha_2 = 0$. If one chooses $\beta \neq |\vec{\alpha}|$, then the fixed point structure consists of an isolated critical point, UV in nature if $|\beta| < |\vec{\alpha}|$ and IR if $|\beta| > |\vec{\alpha}|$. The $\beta = -\alpha_3$ gauge choice corresponds to the constant shift of our angular scalar field $\alpha \to \alpha + c$. The critical point of the superpotential occurs at $r = 0$ with $V$ and $\sigma$ free to take on any value. Thus, we identify $V$ and $\sigma$ as linear combinations of the 5D dilaton and axion. Since $\sigma$ never appears in the scalar potential, either at the fixed point or away from it, we can specify $\sigma$ as the axion. Furthermore, turning on $r$ and $\alpha$ corresponds to turning on 3-form flux dual to the SU(3) singlet operators coming from the $10, \bar{10}$, i.e. gaugino mass terms (modulus dual to $r$, phase dual to $\alpha$).

The pre-potential is written in terms of an SU(2) phase, $Q^s$, and a superpotential, $W$, in the following way:

$$P^s = \sqrt{\frac{3}{2}}\,W Q^s\,. \tag{3.5}$$

Our gauge choice gives us the superpotential

$$W = \left(1+\frac{r^2}{V}\right), \tag{3.6}$$

and the SU(2) phase

$$Q^s = \frac{1}{V+r^2}\left(-2r\sqrt{V}\sin\alpha, -2r\sqrt{V}\cos\alpha, V-r^2\right). \tag{3.7}$$

With this gauging and parameterization, the full scalar potential has a very simple form:

$$\mathcal{V} = -6 + \frac{3r^4}{V^2} - \frac{3r^2}{V}\,. \tag{3.8}$$

Our AdS-sliced domain wall ansatz is:

$$ds^2 = e^{2U(z)} ds^2_{AdS4} + dz^2\,, \tag{3.9}$$



JHEP10(2005)094

$$V = V(z) \tag{3.10}$$
$$\sigma = \sigma(z) \tag{3.11}$$
$$r = r(z) \tag{3.12}$$
$$\alpha = \alpha(z)\,. \tag{3.13}$$

We can now use the machinery of [6 − 8] to calculate the flow equations for the warp factor and the hyper-scalars from the vanishing of the fermionic supersymmetry variations. The function,

$$\gamma = \sqrt{1 - \frac{\lambda^2 e^{-2U}}{g^2 W^2}}\,, \tag{3.14}$$

is often used to simply express the resulting equations. Throughout this paper we use conventions such that lower-case Greek indices $(\mu, \nu)$ refer to bulk space-time, analogous lower-case latin indices (m,n) refer to space-time coordinates along the domain wall, lower-case Latin indices from later in the alphabet (r,s. . . ) refer to SU(2) structure, and upper-case Latin indices refer to hyperscalars or the associated quaternionic geometry. Quantities expressed as an SU(2) triplet (e.g. $P^r$) can be re-expressed as a $2\times 2$ matrix in the usual way:

$$P_i^{\ j} \equiv i(\sigma_r)_i^{\ j} P^r\,. \tag{3.15}$$

For a BPS solution we require the fermionic supersymmetry variations to vanish. In the following expressions, $\mathcal{D}_\mu$ is the total covariant derivative (including both gravity and gauge structure), $f_X^{iA}$ is the quaternionic vielbein, and $\mathcal{N}^{iA} = \frac{\sqrt{6}}{4} f_X^{iA} K^X(q)$, where $K^X(q)$ is the gauged Killing vector. The fermionic SUSY variations are then

$$\delta\psi_{\mu i} = \mathcal{D}_\mu\, \varepsilon_i - \frac{i}{\sqrt{6}}\, g\, \gamma_\mu\, P_i^{\ j} \varepsilon_j\,, \tag{3.16}$$
$$\delta\zeta^A = \frac{i}{2} f_X^{iA} \gamma^\mu (\partial_\mu q^X) \varepsilon_i - g \mathcal{N}^{iA} \varepsilon_i\,. \tag{3.17}$$

We use the residual supersymmetry projector of [8]:

$$i\gamma_5 \varepsilon_i = [A(r) Q_i^{\ j} + B(r) M_i^{\ j}] \varepsilon_j\,, \tag{3.18}$$

where $M^r$ is an SU(2) phase which depends on the scalar fields and is orthogonal to $Q^r$ (i.e. $Q^r M^r = 0$). This is the most general ansatz for residual supersymmetry consistent with an AdS-sliced domain wall. It was found in [8] that the quantities $A(r), B(r)$ are constrained up to signs by consistency and integrability constraints. There is an additional consistency constraint on the choice of $M^r$ that is derived in [6, 12], using $\theta^r = -\gamma Q^r + \sqrt{1-\gamma^2} M^r$:

$$[\theta, \nabla_z\, \theta] = -\sqrt{\frac{2}{3}} g\, [\theta, P]\,, \tag{3.19}$$

and it should be noted that the conventions of [6] set $g = \sqrt{3/2}$. Writing this matrix equation in terms of SU(2) triplets we find

$$\theta^r \nabla_z\, \theta^s \varepsilon^{rst} \sigma^t = -\sqrt{\frac{2}{3}}\, g\, \theta^r P^s \varepsilon^{rst} \sigma^t\,. \tag{3.20}$$

Once this consistency condition is satisfied, the BPS equations will tell us the evolution of the scalar fields and the geometry.

Integrability of the gravitino variation condition along the wall ($\delta\psi_m = 0$) and transverse to the wall ($\delta\psi_5 = 0$) give two different expressions involving $U'$. These may be solved for $U'$ and the previously unknown function $A$:

$$U' = \pm\gamma(z)\,|gW| \tag{3.21}$$
$$A = \mp\gamma(z)\,, \tag{3.22}$$

where the sign choice in these two equations is correlated. Note that consistency of the projector equation (3.36) determines the magnitude but does not fix the sign of the function $B$:

$$B = \pm\sqrt{1-\gamma^2}\,, \tag{3.23}$$

where this upper/lower sign choice is independent of that in $A$.

As long as we satisfy equation (3.20), vanishing of the hyperini variations will now give us the flow equations of the hyper scalars, according to the formulas of [7, 8] (equivalent expressions can be found in [6] using somewhat different language, and a nice summary and dictionary between the two languages can be found in [12]). We first write down the general form of the flow equations with an arbitrary projector, $M^r$. We will later make a specific choice and prove its consistency for the Super Janus gauging.

Let $q^X$ denote the $X^{\text{th}}$ hyper scalar, $g$ the gauge coupling, $R^t_{XY}$ the SU(2) curvature (see [7] for the curvature formulas). The general flow equations are

$$q'^X = 3g\left(A\,g^{XY} + 2B\,\varepsilon^{rst}M^r Q^s R^{tXY}\right)\partial_Y W\,. \tag{3.24}$$

The signs of the $A$ and $B$ functions can be determined by demanding that the warp factor has a turning point at the domain wall (matching to Janus) and that the Killing spinor equation be continuous across the domain wall. The turning point condition gives

$$\gamma(0) = 0 \tag{3.25}$$
$$A = -\mathrm{sgn}(z)\gamma\,. \tag{3.26}$$

Continuity of the Killing spinor tells us that $B$ does not flip sign at the wall; thus,

$$B = \sqrt{1-\gamma^2}\,. \tag{3.27}$$

This procedure is analogous to that followed in [1, 2] for the original Janus solution. In those papers, geodesic completeness of the space-time demanded that the warp factor be analytically continued as an even function of $z$ to the other side of the wall. (N.B: In those papers, the warp factor was denoted by $A(z)$ rather than $U(z)$). We believe that this symmetry of the original Janus solution was somehow accidental, as Super Janus is slightly asymmetric, and there is no trace of this symmetry in the dual of either theory.



JHEP10(2005)094

Before fixing the projector phase, $M^r$, the complete set of flow equations for our parameterization is given by the following:

$$U'(z) = \pm g W \gamma = \pm \sqrt{g^2 \left(1 + \frac{r^2}{V}\right)^2 - \lambda^2 e^{-2U}} \tag{3.28}$$

$$V'(z) = 6g\, r \left[ \pm r\gamma - \frac{1}{V + r^2} \left( \sqrt{V}\sqrt{1-\gamma^2}(-2M^3 r\sqrt{V} + \right.\right. \tag{3.29}$$

$$\left.\left. + (M^2 \cos\alpha + M^1 \sin\alpha)(r^2 - V)) \right) \right] \tag{3.30}$$

$$\sigma'(z) = \frac{3g\, r\sqrt{(1-\gamma^2)}}{\sqrt{V}(r^2 + V)} \left( M^2(-3r^4 + 5r^2 V + 2V^2)\sin\alpha + \right. \tag{3.31}$$

$$\left. + 2\cos\alpha \left( M^1 (r^2 - V)^2 + M^3 r^3 \sqrt{V} \sin\alpha \right) \right) \tag{3.32}$$

$$r'(z) = 3g\, r \left( \mp\gamma - \frac{r\sqrt{1-\gamma^2}}{(r^2 + V)\sqrt{V}} \left( -2M^3 r\sqrt{V} + \right.\right. \tag{3.33}$$

$$\left.\left. + \left(M^2 \cos\alpha + M^1 \sin\alpha\right)\left(r^2 - V\right) \right) \right) \tag{3.34}$$

$$\alpha'(z) = \frac{3g\, r\sqrt{(1-\gamma^2)}}{2\sqrt{V}(r^2 + V)} \left( \left(2M^1 \cos\alpha - 3M^2 \sin\alpha\right)\left(r^2 - V\right) + M^3 r\sqrt{V} \sin 2\alpha \right). \tag{3.35}$$

The sign choices come from the projector function $A$, thus, we must choose the upper sign for $z > 0$ and the lower sign for $z < 0$. We again emphasize that this is consistent and smooth because Janus-like solutions require $U' \sim \gamma \to 0$ as $z \to 0$, and all terms that flip sign at the domain wall are linear in $\gamma$ and thus continuous.

We follow the strategy of [8], first we pick $\alpha(0) = 0$ as part of our initial conditions, then an easy guess for our projector phase is

$$M^s = (0, Q^3, -Q^2) = \frac{1}{V + r^2}(0, -r^2 + V, 2r\sqrt{V}). \tag{3.36}$$

Now we must check equation (3.20). Since the Pauli matrices are linearly independent, we may drop them and regard (3.20) as three separate equations. Then, because $\theta^1 = P^1 = 0$, only the $t = 1$ component is non-trivial. The action of the covariant derivative on $\theta^r$ may be written as

$$\nabla_z \theta^r = q^{X'} \nabla_X \theta^r + U' \partial_U \theta^r. \tag{3.37}$$

With $\alpha(0) = 0$ and the projector (3.36), we find $\alpha' = \sigma' = 0$, and

$$V'(z) = 6g \left( \pm r^2 \gamma + r\sqrt{V}\sqrt{1-\gamma^2} \right) \tag{3.38}$$

$$r'(z) = 3g \left( \mp r\, \gamma + \frac{r^2}{\sqrt{V}}\sqrt{1-\gamma^2} \right). \tag{3.39}$$



JHEP10(2005)094

This greatly simplifies the task of checking consistency:

$$\theta^2 \nabla_z \theta^3 - \theta^3 \nabla_z \theta^2 = -\left(1 + \frac{r^2}{V}\right) \sqrt{1-\gamma^2} \tag{3.40}$$

$$\theta^2 P^3 - \theta^3 P^2 = \frac{1}{g}\sqrt{\frac{3}{2}} \left(1 + \frac{r^2}{V}\right) \sqrt{1-\gamma^2}\,, \tag{3.41}$$

so we see that (3.20) is satisfied. Since $M^r$ has three components, and there are 3 independent consistency equations, this completely specifies the projector phase.

We now have a system of three coupled, non-linear, ordinary differential equations for the warp factor and two running scalars (3.28), (3.38), (3.39). We will solve these numerically in the next subsection.

If one were to choose purely the upper sign in equations (3.28), (3.38), (3.39), then one obtains the solution of [8], which is nakedly singular at the domain wall. Super Janus avoids this by the requirement that the warp factor have a turning point at the domain wall, which we enforce through a careful choice of initial conditions. We believe the initial conditions chosen in [8] were such that they guaranteed a curvature singularity. Indeed, we will find that only a narrow range of parameter space allows a turning point.

### 3.2 Numerics

We enforce the turning point condition with a suitable choice of initial condition for $V$. Setting equation (3.28) to zero at $z = 0$, we obtain

$$V(0) = \frac{g\, r^2(0)}{\pm\lambda\, e^{-U_0} - g}\,. \tag{3.42}$$

The scalar field $V$ must be strictly positive[1], so we must choose the plus sign, as there is no positive value of $\lambda\, e^{-U_0}$ that gives positive $V$ when the minus sign is chosen. With the plus sign choice, we must still require $\lambda\, e^{-U_0} > g$ for positivity of $V$. An additional constraint on the initial condition for the warp factor comes from the requirement that the turning point be a minimum: $U'' > 0$. We may easily calculate $U''$ using the BPS equations to replace $r'$, $V'$, and $U'$ as needed. The result is a surprisingly simple expression:

$$U''(z) = \lambda^2 e^{-2U(z)} - \frac{6g^2\, r^4}{V^2} - \frac{6g^2\, r^2}{V}\,. \tag{3.43}$$

Imposing the boundary condition for $V$ gives us

$$U''(0) = \lambda\, e^{-U_0}\left(6g - 5\lambda e^{-U_0}\right). \tag{3.44}$$

Positivity of this expression requires $\lambda\, e^{-U_0} < \frac{6}{5}\, g$. Both constraints together limit us to a very narrow band:

$$g < \lambda\, e^{-U_0} < \frac{6}{5}\, g\,. \tag{3.45}$$

[1]Strictly speaking, $V$ must be strictly of one sign. In many theories, it plays the role of the volume of a Calabi-Yau manifold, so it is conventional to choose it positive. See [7] for more details.

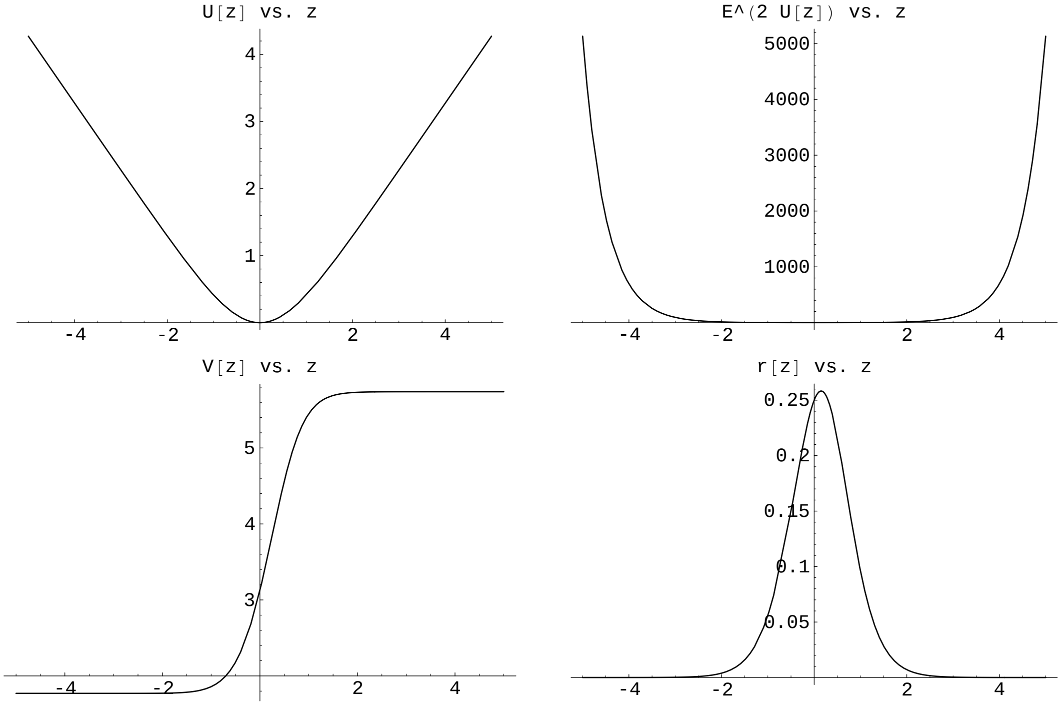


**Figure 1:** Numerical results for a typical choice of parameters: $\lambda\, e^{-U_0} = 1.02, r_0 = 0.25$.

We may now numerically integrate equations (3.28), (3.38), (3.39). As long as our initial conditions are within the critical range, we find the scalar $V$ develops a profile very much like the 5D dilaton in Janus. By adjusting $r(0)$ (arbitrarily) and $\lambda\, e^{-U_0}$ (within criticality), we may adjust the average value of $V$ as well as the split between the two different asymptotic values of $V$.

In figure 1 we plot the numerical results for a typical choice of parameters: $\lambda\, e^{-U_0} = 1.02, r_0 = 0.25$.

### 3.3 Asymptotic behavior

Using our knowledge from the numerics of the asymptotic behavior of the fields $V$ and $r$ and the warp factor $U$, we can use the flow equations (3.38), (3.39) to determine the subleading behavior for matching against the field theory dual. The asymptotic flow equations are

$$U'(z) \underset{z\to\pm\infty}{\longrightarrow} \pm g \tag{3.46}$$

$$r'(z) \underset{z\to\pm\infty}{\longrightarrow} \mp 3g\, r \tag{3.47}$$

$$V'(z) \underset{z\to\pm\infty}{\longrightarrow} 6g\left(\pm r^2 + r\sqrt{V}\,\frac{\lambda\, e^{-U}}{1+\frac{r^2}{V}}\right). \tag{3.48}$$

Setting $g = 1$ as we did in the numerics, equations (3.46), (3.47) may be easily solved to yield

$$e^{U(z)} \underset{z\to\pm\infty}{\longrightarrow} c_U\, e^{\pm z} \tag{3.49}$$



JHEP10(2005)094

$$r(z) \underset{z\to\pm\infty}{\longrightarrow} c_r\, e^{\mp 3z}\,, \tag{3.50}$$

where we have left out a constant term in $r(z)$ because numerics show $r \underset{z\to\pm\infty}{\longrightarrow} 0$. Plugging these back into equation (3.48) and Taylor expanding in $e^{-z}$ we find

$$V' \to 6\left(c_r^2\, e^{-6z} + c_r\, c_U \lambda \sqrt{V} e^{-4z}\right), \tag{3.51}$$

which gives us

$$V \to V_\pm - c_r \frac{3\lambda\, c_U}{2} \sqrt{V_\pm} e^{-4z}\,. \tag{3.52}$$

This behavior is what we expect for $V$ being dual to a dimension 4 operator with source term and $r$ to a dimension 3 operator with no source term, as expected if $V$ is the 5D dilaton and $r$ is related to 3-form flux (or gaugino mass terms). In order to read off the vevs of the corresponding operators in the dual gauge theory we need to determine the two integration constants $c_U$ and $c_r$, which only get fixed by the IR behavior of the solution and hence at the moment can only be determined from our numerical solution.

## 4. Conclusion

We have found a numerical solution for a supersymmetric domain wall in 5D, $\mathcal{N}=2$ gauged supergravity which is supported by two hyperscalars. The BPS equations allow us to determine analytically that the asymptotic behavior of these hyperscalars is appropriate for the duals of a dimension 4 and a dimension 3 operator. The uplift of this solution to type-IIB supergravity in 10D is the Super Janus solution, dual to the gauge theory of [3] with partial SUSY restoring counter-terms. The 10D domain wall solution will be supported by dilaton, 5-form flux, and 3-form flux dual to gaugino mass.

The existence of uplifts to 10D is guaranteed by the embedding in [7] of the $\mathcal{N}=2$ theory with a single hypermultiplet into the FGPW solution [13] of the $\mathcal{N}=8$ theory, where the uplift formulas of [4] apply. Unfortunately, to realize the uplift one must understand how the hyperscalars of the $\mathcal{N}=2$ theory sit inside the 27-bein of the $E_{6(6)}/\mathrm{USp}(8)$ coset of the $\mathcal{N}=8$ theory, which is far from straightforward. Solving this would immediately give the uplift of Super Janus. Another potential route is to solve the 10D equations of motion directly with some suitable ansatz deforming the $\mathrm{AdS}_5 \times S^5$ geometry to something asymptotically $\mathrm{AdS}_5$ crossed with an internal space possessing only SU(3) isometry.

It would also be interesting to understand the critical range of parameters in Janus and Super Janus from the perspective of the boundary gauge theories. There is no obvious reason for the gauge theory to care about the value of the jump in the coupling constant. Finding the importance of this critical range in the gauge theory would shed light on the Janus and Super Janus solutions.

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